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Common final exam for Math 117, May 2nd, 2024.

YOUR NAME: ____________________ SECTION: ____________________

INSTRUCTOR: ____________________

Directions:

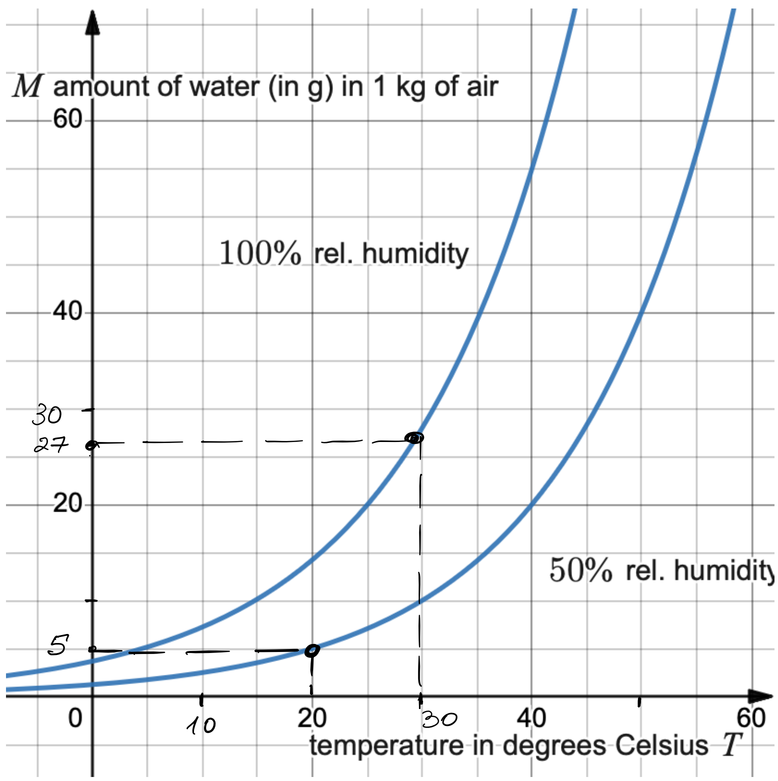
- Print your name, section number and your instructor's name on this page in the space provided.
- This exam has 15 questions. Please check that your exam is complete.
- You have two hours to complete this exam. It will be graded out of 120 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator and the list of equations attached at the end of the exam.
- When using decimals round your answers to three decimal places.
- You're not allowed to use notes, books, any internet resources, or electronic devices (except for a calculator).
- You may not communicate with anyone besides the instructor during this exam.

Problem	Score
1	/6
2	/12
3	/6
4	/8
5	/8
6	/6
7	/6
8	/8
9	/10
10	/8
11	/6
12	/8
13	/8
14	/10
15	/10
	/120

Good luck!

1. (Points: 6)

The figure below shows the mass of water in air, in grams of water per kilogram of air, as a function of air temperature in °C, for two different levels of relative humidity.



(a) Find the mass of water in 1 kg of air at $30°C$ if the relative humidity is 100%. Include units

27 gram of water in 1 kg of air (2 points)

(b) How much water in grams is in a room containing 300 kg of air if the relative humidity is 50% and the temperature is 20°C?

5 gram of water in 1 kg of air.

In 300 kg of air : $5 \cdot 300 = 1500$ gram of water

(4 points)

2. (Points: 12)

In a college meal plan you pay a membership fee; then all your meals are at a fixed price per meal.

3 points

(a) If 90 meals cost \$1005 and 140 meals cost \$1205, write a linear function that describes the cost of a meal plan, C, in terms of the number of meals, n.

$n \quad C(n)$

$$C(n) = m \cdot n + b$$

$$m = \text{slope} = \frac{1205 - 1005}{140 - 90} = \frac{200}{50}$$

$$m = 4$$

$$C(n) = 4n + b \leftarrow (90, 1005)$$

$$1005 = \underbrace{4 \cdot 90}_{} + b$$

$$1005 = 360 + b$$

$$1005 - 360 = b \Rightarrow b = 645$$

3 points

(b) What is the cost per meal and what is the membership fee?

$$C(n) = 4 \cdot n + 645$$

cost per meal

membership fee

3 points

(c) Find the cost for 120 meals. $n = 120$

$$C(120) = \underbrace{4 \cdot 120}_{} + 645 =$$

$$= 480 + 645 = 1125$$

3 points

(d) Determine the maximum number of meals you can buy on a budget of \$1285.

$$1285 = 4 \cdot n + 645$$

$$-645 \qquad -645$$

$$\frac{640}{4} = \frac{4 \cdot n}{4} \Rightarrow n = 160 \text{ meals}$$

3. (Points: 6)

For the function $f(x)$ given below find the value of the inverse function $f^{-1}(3)$. Give the exact answer or round your answer till three decimal places.

$$f(x) = \frac{4x+3}{2-5x}.$$

$$\underset{\substack{\downarrow \\ \text{output}}}{y} = f(\underset{\substack{\downarrow \\ \text{input}}}{x}) \iff \underset{\substack{\downarrow \\ \text{output}}}{x} = f^{-1}(\underset{\substack{\downarrow \\ \text{input}}}{y})$$

$$y = \boxed{3 = \frac{4x+3}{2-5x}} \Rightarrow 3\cdot(2-5x) = 4x+3$$

2 points

$$6 - 15x = 4x + 3$$

$$+15x \qquad +15x$$

2 points

$$6 = 19x + 3$$

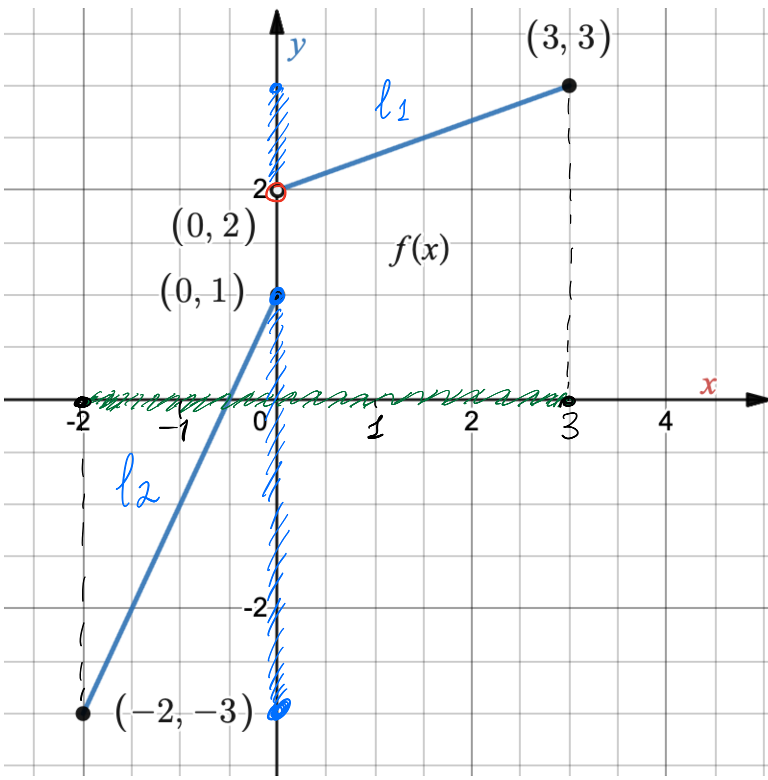
$$-3 \qquad -3$$

2 points

$$\frac{3}{19} = \frac{19x}{19} \Rightarrow \boxed{x = \frac{3}{19}} = f^{-1}(3)$$

4. (Points: 8)

Use the graph of f below to answer the following questions.



ℓ_1: slope $= m = \frac{3-2}{3-0} = \frac{1}{3}$

$b = 2$

$\boxed{y = \frac{1}{3}\cdot x + 2}$

ℓ_2: slope $= m = \frac{1+3}{0+2} = \frac{4}{2}$

$b = 1$ $\quad m = 2$

$\boxed{y = 2\cdot x + 1}$

(a) Fill in the blanks to give a piecewise-defined expression for f.

$$f(x) = \begin{cases} 2\cdot x + 1, & -2 \le x \le 0 \\ \frac{1}{3}x + 2, & 0 < x \le 3 \end{cases} \qquad (1)$$

2 points

2 points

(b) Give the domain and range of f.

Domain: $-2 \le x \le 3$ 2 points

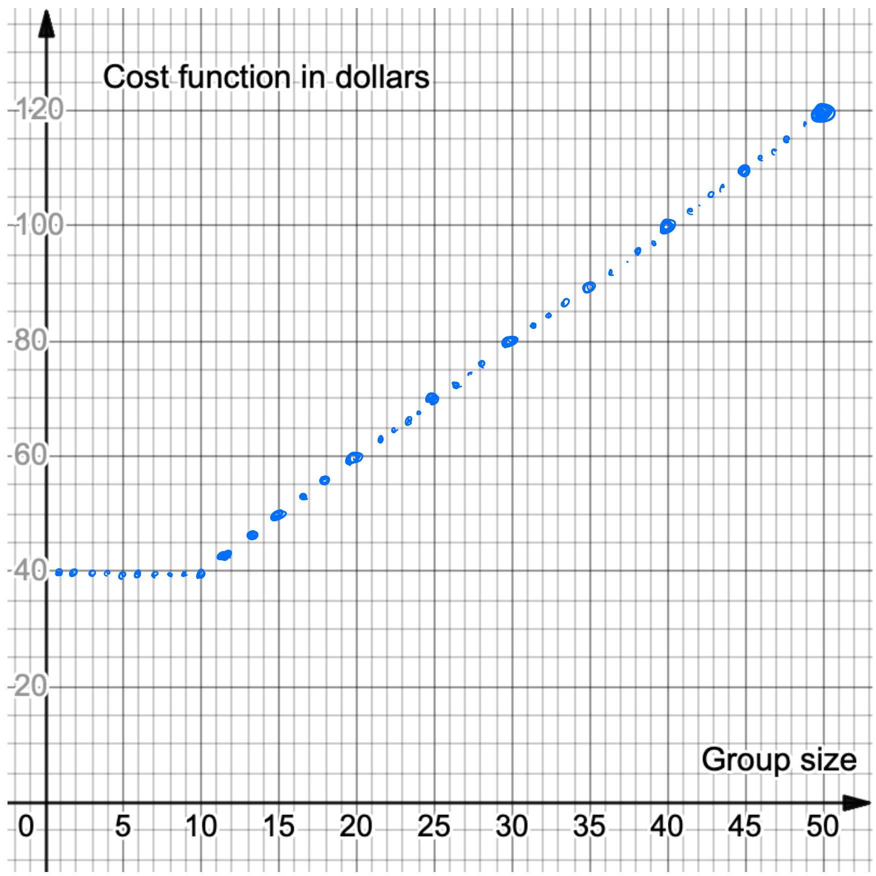
Range: $-3 \le y \le 1$, $2 < y \le 3$ 2 points

5. (Points: 8)

A museum charges \$40 for a group of 10 or fewer people. A group of more than 10 people must, in addition to the \$40, pay \$2 per person for the number of people above 10. For example, a group of 12 pays \$44 and a group of 15 pays \$50. The maximum group size is 50.

(a) Draw a graph that represents this situation.

4 points



(b) What are the domain and range of the cost function?

2 points

Domain: group size: 0, 1, 2, 3, 4, 5 . . 50
integers

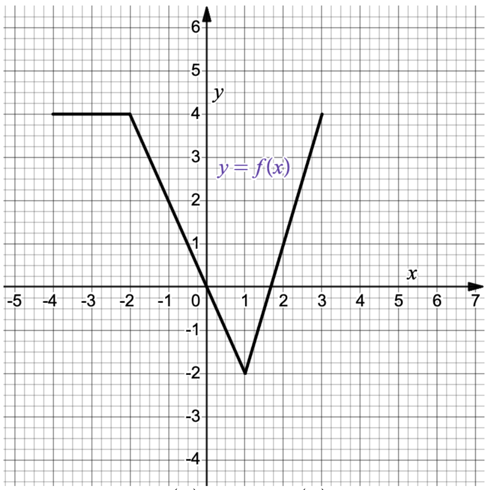
2 points

Range: cost function: 40, 42, 44, 46 120

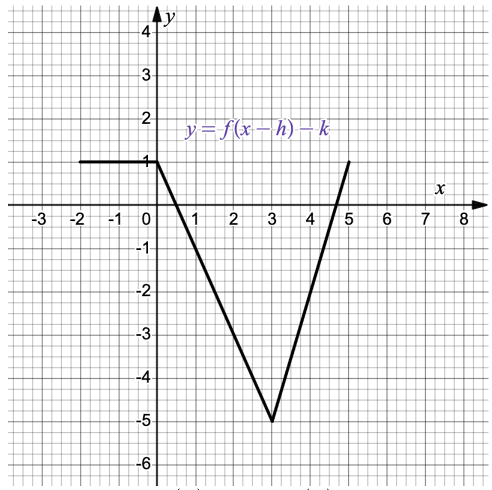
answer: $0 \leq x \leq 50$, $40 \leq y \leq 120$ – should be accepted as the correct one

6. (Points: 6)

The Figure (a) below shows the graph $y = f(x)$.



(a) Figure (a)



(b) Figure (b)

Find a formula in terms of f for the graph of the function in Figure (b). Your formula should be of the form $y = f(x - h) + k$ for appropriate constants h and k.

$y = f(x-2) - 3$

$h = 2$, $k = -3$

7. (Points: 6) Let, $C = C(F) = \frac{5}{9}(F - 32)$ where C is temperature in degrees Celsius and F is in degrees Fahrenheit. The temperature,

$$F = F(n) = 68 + \frac{10}{2 + n^2},$$

in degrees Fahrenheit of a room is a function of the number, n, of hours that the air conditioner has been running. Find $C(F(5))$. Round your answer to two decimal places and give appropriate units.

3 points

$F(5) = 68 + \frac{10}{2+5^2} = 68.37037037\ ^\circ F$

3 points

$C(68.37037037) = \frac{5}{9}(68.37037037 - 32) = 20.206\ ^\circ C$

8. (Points: 8) The table below shows the concentration $C = f(t)$ (in millimoles per liter) of the chemical phenolphthalein in solution as a function of time t in seconds. By evaluating successive rates of change determine if f is concave up or concave down?

t sec	1.5	2.5	3.5	4.5
C	2.117	3.490	5.745	9.488

2 points for each calculation

$\frac{3.490 - 2.117}{2.5 - 1.5} \Downarrow 1.373$

$\frac{5.745 - 3.49}{3.5 - 2.5} \Downarrow 2.255$

$\frac{9.488 - 5.745}{4.5 - 3.5} \Downarrow 3.743$

2 points for the conclusion

Since successive rates of change are increasing from left to right $f(t)$ is concave up

9. (Points: 10)

A ball is thrown into the air. Its height (in feet) t seconds later is given by

$$h(t) = 80t - 16t^2.$$

$$h(t) = \underbrace{-16}_{a}t^2 + \underbrace{80}_{b}t + \underbrace{0}_{c}$$

4 points

(a) Write the formula for $h(t)$ in the vertex form. $h(t) = a(t-h)^2 + K$

$$h = \frac{-b}{2 \cdot a} = \frac{-80}{2 \cdot (-16)} = 2.5$$

$$K = h(2.5) = 80 \cdot 2.5 - 16 \cdot (2.5)^2 = 100$$

$$\boxed{h(t) = -16(t-2.5)^2 + 100}$$

(b) Using the vertex formula determine the time required to reach the peak altitude from the ground.

3 points

$t = 2.5$ seconds

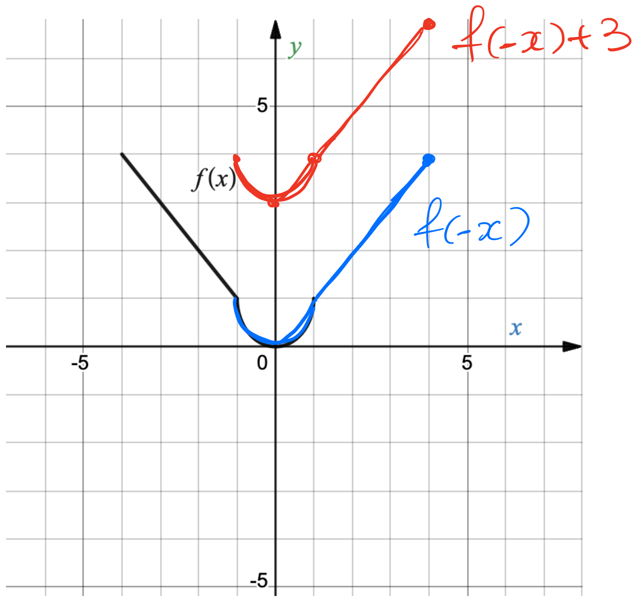
(c) Calculate the maximum height of the ball relative to the ground.

3 points

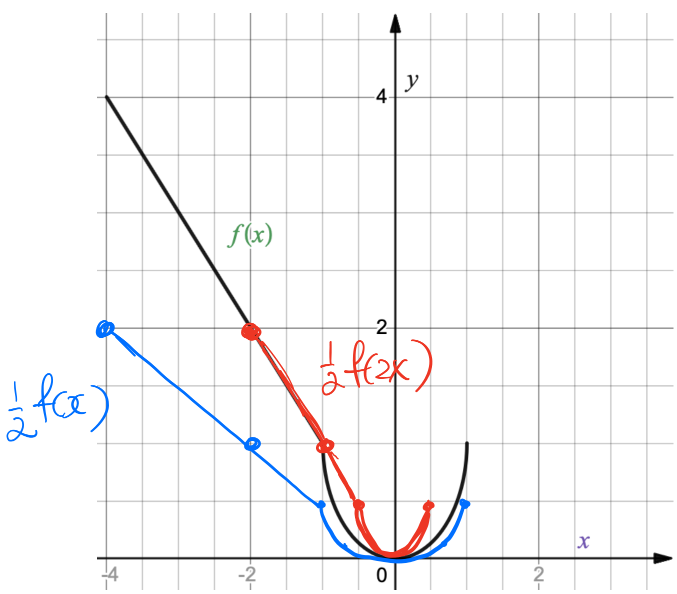
$h_{max} = 100$ feet

10. (Points: 8) Graph the following transformations of the function $f(x)$ on the same axes.

(a) $y = f(-x) + 3$



(b) $y = \frac{1}{2} f(2x)$

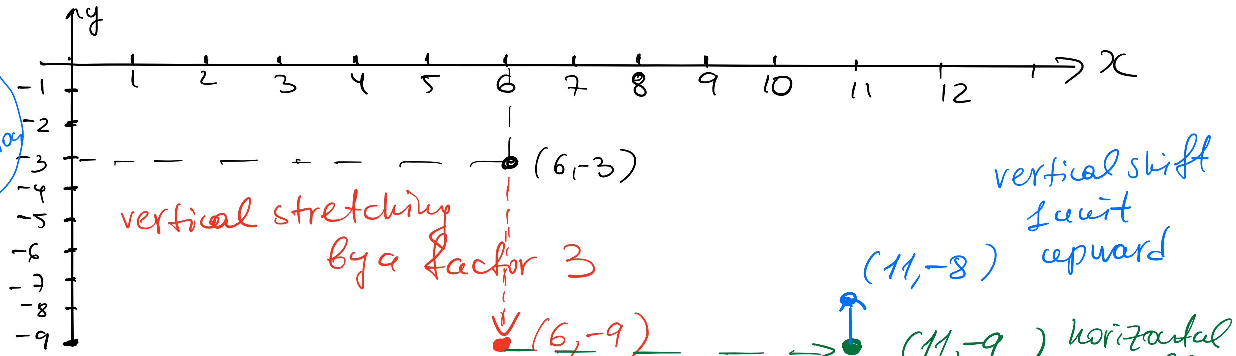


11. (Points: 6) The point $(6, -3)$ is on the graph of $g(x)$.

What point must be on the graph of the function $3g(x-5)+1$?

answer: $(11, -8)$

2 points each transformation



12. (Points: 8) The total cost $C(n)$ in dollars for a producer to manufacture n units of a good is given by

$$C(n) = 500 + 5n.$$

The average cost of producing n units is

$$a(n) = \frac{C(n)}{n}. = \frac{500+5n}{n} = \frac{500}{n} + 5$$

2 points

(a) Evaluate and interpret the economic significance of $C(1000)$.

$C(1000) = 500 + 5 \cdot 1000 = \$5500$ – cost of producing 1000 units

3 points

(b) Evaluate:

i. $a(1000) = \frac{500}{1000} + 5 = 5.5$

ii. $a(10000) = \frac{500}{10000} + 5 = 5.05$

iii. $a(100000) = \frac{500}{100000} + 5 = 5.005$

3 points

(c) Based on part (b), what trend do you notice in the values of $a(n)$ as n gets large? Explain this trend in economic terms.

$a(n) \to 5$ as $n \to \infty$

13. (Points: 8)

For the rational function given below find all finding zeros, vertical and horizontal asymptotes.

$$y = \frac{2x + 3}{x^2 - 2x - 3} = \frac{2x+3}{(x-3)(x+1)}$$

zeros : $2x+3=0$

$2x=-3$

$x=-\frac{3}{2}$

2 points

vertical asymptotes:

$(x-3)\cdot(x+1)=0$

$\Downarrow \quad \Downarrow$

$0 \quad 0$

$x=3 \quad x=-1$

3 points

horizontal asymptote:

$y \approx \frac{2x}{x^2} = \frac{2}{x} \to \boxed{0=y}$

as $x \to \pm\infty$ $\boxed{y=0}$

3 points

14. (Points: 10)

Find a possible formula for a fourth degree polynomial function $g(x)$ that has a double zero at $x = 4$ and $g(5) = 0$, $g(-1) = 0$ and $g(0) = 4$.

5 points

$g(x) = a\cdot(x-4)^2\cdot(x-5)\cdot(x+1) \leftarrow \quad g(0)=4$

$4 = a\cdot(0-4)^2\cdot(0-5)\cdot(0+1)$

$4 = a\cdot(-4)^2\cdot(-5)\cdot(1)$

$4 = a\cdot(16)\cdot(-5)$

$\frac{4}{(16)(-5)} = a$

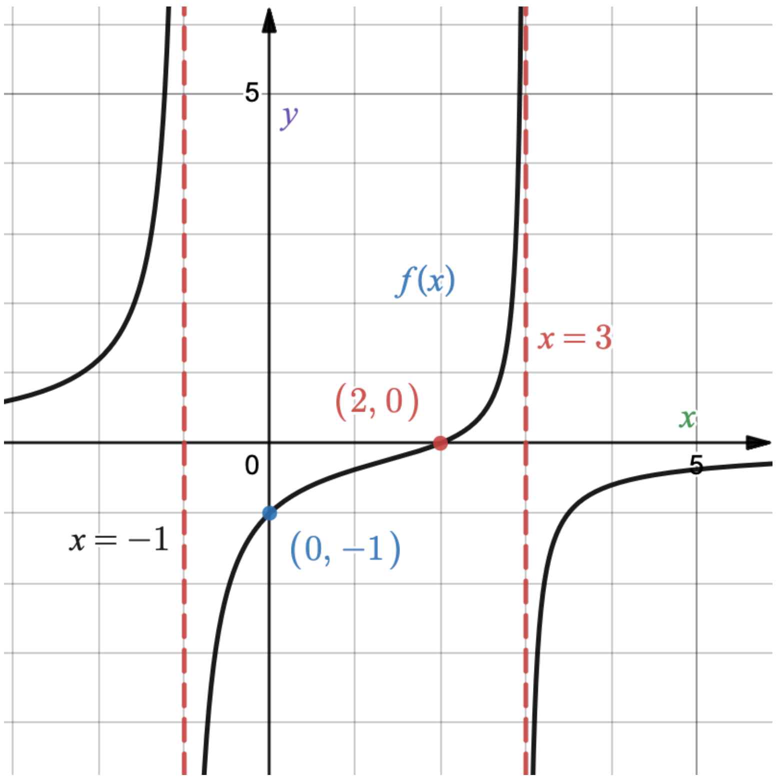
$-\frac{1}{20} = a$

5 points

$$\boxed{g(x) = -\frac{1}{20}(x-4)^2(x-5)(x+1)}$$

15. (Points: 10)

The function f is a rational function. Its graph is shown below. Give a possible formula for $f(x)$.



5 points

$$f(x) = A \frac{x-2}{(x+1)\cdot(x-3)} \leftarrow (0,-1)$$

$$-1 = A \cdot \frac{(0-2)}{(0+1)(0-3)}$$

5 points

$$-1 = A \cdot \frac{(-2)}{(1)\cdot(-3)}$$

$$A = -\frac{3}{2}$$

$$\boxed{f(x) = -\frac{3}{2} \cdot \frac{x-2}{(x+1)(x-3)}}$$

HAVE A NICE SUMMER!

Formulas

Average rate of change: $\dfrac{f(b) - f(a)}{b - a}$

Slope-intercept form: $y = b + mx$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Quadratic function: $y = ax^2 + bx + c$

Factored form: $y = a(x - r)(x - s)$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vertex form: $y = a(x - h)^2 + k$

Power function $y = kx^p$

Directly proportional: $y = kx$

Inversely proportional: $y = \dfrac{k}{x}$

Factored form of a polynomial: $p(x) = c(x - a_1)(x - a_2)\cdots(x - a_n)$